

# Picking a Present Value Estimate of Future Earnings: The Role of Simulation

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## Abstract

In a classic paper, Dulaney (1987) proposes a historical simulation-based method for evaluating measures of the present value of future earnings. This method compares a given ex ante estimate of present value with an ex post simulated value, in each time period. A key issue is the interpretation of what it means to have a good fit, when matching historical simulated present values. With best fit defined in standard statistical terms, I find that the total offset approach – whereby projected growth in wages is assumed equal to the projected interest rate – works best in the examples considered here and in Dulaney (1987). This finding violates convention as most forensic economists implicitly allow a gap between projected wage growth and the interest rate, when estimating present value. It does, however, jibe with the absence of a statistically significant long-run gap between U.S. annual wage growth and the interest rate.

# 1 Introduction

When evaluating wage losses in cases of personal injury or death, the selection of projection and discounting techniques is a central issue. This was the case 20+ years ago when John Ward and Gerald Olson inaugurated *The Journal of Forensic Economics* (JFE), with an opening essay in which they called attention to the problems of projection and discounting. There is no lack of methods: to the contrary, there are numerous approaches to projection and discounting, as surveyed in Dulaney (1987) and Brush (2002, 2004). However, in any particular application the forensic economist may find it hard to choose among existing methods.

In their opening essay in the first JFE issue, Ward and Olson (1987) suggest a role for simulation in evaluating the performance of projection and discounting methods. In that same JFE issue, Dulaney (1987) proposes an innovative method of simulation, which he calls historical simulation, for assessing present valuation methods. He compares various estimation methods, applied to the value of future earnings, and identifies one such method – the base period method – as being best in terms of simulation performance.

In the present work I reconsider the comparison of present value estimation methods, via Dulaney’s historical simulation. The simulation compares a given ex ante estimate of present value with an ex post simulated value, in each time period. In the spirit of Dulaney (1987), the method that best

matches the simulated values is deemed most desirable. A key issue is the interpretation of what it means to have a good fit, when matching historical simulated present values. In the present work I define best fit in standard statistical terms, via mean squared error and also mean absolute error. By contrast, Dulaney (1987) measures fit via the maximum of computed gaps between estimated and simulated values: this approach focuses on extreme events and hence is non-standard from the standpoint of conventional statistical/forecasting methodology.

For estimating present value I find that the total offset approach – whereby projected growth in wages is assumed equal to the projected interest rate – works best in the examples considered here and in Dulaney (1987) and in the recent update of his work by Brush (2004). This finding violates convention as most forensic economists implicitly allow a gap between projected wage growth and the interest rate, when estimating present value, see Brush (2003) and Brookshire and Slesnick (1999).<sup>1</sup>

Consistent with the simulation-based superiority of the total offset estimate of present value, I find no statistically significant historical difference between long-run U.S. wage growth and the interest rate, in the period 1953-2008. The net discount rate, that embodies such differences, fluctuates over time. However, using a battery of tests I fail to reject the hypothesis that its long-run mean value equals 0. These formal statistical results echo some informal arguments made by Schwartz (1997).

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<sup>1</sup>See also Ireland (2006) for recent discussion.

## 2 Estimation Methods

As in Dulaney (1987) and Brush (2004), I consider four methods of estimating the present value of future income. The starting point is the estimated value formula:

$$EPV = \sum_{i=1}^n Y \left( \frac{1+W}{1+R} \right)^i \quad (1)$$

where  $Y$  is base year earnings,  $W$  is the projected annual growth in nominal income,  $R$  is the projected nominal rate of interest, and  $n$  is the number of future periods under consideration. As in Dulaney (1987) I set  $n$  equal to 20.

Four standard estimation methods, each a special case of equation 1, are:

1. Base Year Projection Approach:  $W$  equals the earnings growth in the base period, and  $R$  equals the nominal interest rate in the base period.
2. Base Period Projection Approach:  $W$  is an average of earnings growth rates in several periods, ending in the base period, and similarly  $R$  is an average of interest rates in several periods.
3. Historical Period Projection Approach:  $W$  and  $R$  are averages, as in the base period approach, but the average is taken over a fixed historical period.
4. Total Offset Approach: The values of  $W$  and  $R$  are posited to be equal, in which case  $EPV = nY$ .

As in Dulaney (1987), for the Base Period Projection Approach I assume that averaging is done over the most recent 3 years.

To gain further insight into the four different estimation approaches, we can re-write equation 1 as:

$$EPV = \sum_{i=1}^n Y (1 + G)^i \quad (2)$$

where  $G$  represents the growth-discount rate, also called the net discount rate, defined as:

$$G = \frac{1 + W}{1 + R} - 1 \quad (3)$$

The case of total offset is where  $G = 0$ : as an estimation method it is particularly simple since it requires no data to estimate  $W$ ,  $R$ , or  $G$ .<sup>2</sup>

### 3 Historical Simulation

Dulaney (1987) proposes to evaluate estimates of present value in terms of their proximity to historically simulated present value, defined as:

$$SPV = \sum_{i=1}^n Y \left( \frac{1 + W_i}{1 + R_i} \right)^i \quad (4)$$

where  $W_i$  is the actual rate of income growth in year  $i$ , and  $R_i$  is the actual

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<sup>2</sup>See Pelaez (1989) for further discussion.

interest rate in year  $i$ , for each of the years  $i = 1, \dots, n$ . Dulaney (1987) and Brush (2004) interpret  $SPV$  as the “actual” present value of future earnings.  $SPV$  is computed using future information that is unavailable during the base period: it is therefore a sort of “ex post” present value. It is not itself useful as an estimated present value, since it requires information that is unavailable in the base period. However, it suggests an interesting comparison of ex ante estimates of present value to the ex post statistic  $SPV$ .

Dulaney (1987) evaluates ex ante present value estimates  $EPV$  based on their proximity to the ex post statistic  $SPV$ . For this he computes  $EPV$  and  $SPV$  over some historical periods, and compares them in terms of their historical averages and also in terms of their maximum mean absolute difference/error, with maximum computed over the whole sample of relevant data. The latter is not a commonly used measure of forecasting performance, and has the unfortunate property of focusing on extreme events in the sample, hence I avoid it.

In the present work, I focus on a standard measure of forecasting performance, the Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (EPV_i - SPV_i)^2} \quad (5)$$

An attractive property of  $RMSE$  is that it measures the “distance” between  $EPV$  and  $SPV$  using the same type of Euclidean metric that underlies the choice of sample averages as least-distance estimates of wage growth and

interest rates in the Base Period Projection and Historical Period Projection approaches to present value estimation. Also, the squared value of *RMSE* decomposes nicely into bias and variance components – see Section 6.

For forecasting, both RMSE and mean absolute error (MAE) are commonly reported. For the examples in the present work I get the same conclusion, in terms of best-ranking *EPV* methods, using either method, for 5 out of 6 cases. In the remaining case the ranking is a bit different but the conclusion is nearly the same as for the other cases. Hence, I will focus on the RMSE results.

## 4 Data

I apply the *RMSE* measure of discrepancy, between estimated present value *EPV* and simulated present value *SPV*, using the data definitions of Dulaney (1987) and Brush (2004), but with the addition of data for more recent years. The measure of income is U.S. hourly compensation for the business sector: I obtain this from the U.S. department of Labor, Bureau of Labor Statistics, in the online Productivity and Costs tables.<sup>3</sup> The measure of interest rate is the market yield on U.S. Treasuries, 3-year constant maturity, quoted on investment basis: I obtain this from the U.S. Federal Reserve Board, online table H.15.<sup>4</sup>

In the work of Dulaney (1987), the sample period is from 1953 to 1986.

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<sup>3</sup>Series Id:PRS84006103

<sup>4</sup>Unique Identifier: H15/H15/RIFLGFCY03\_N.M

With currently available data, I extend the sample to the year 2008. For hourly earnings, data is quarterly and I annualize the quarterly values via a simple average. I then compute  $W$  as the annual percentage change in hourly wage. For the interest rate, data is monthly and I annualize by averaging monthly rates.<sup>5</sup>

## 5 Results

Consider first the historical period studied by Dulaney (1987). This period spans the years 1953-1986. Dulaney uses the subperiod 1953-1968 to compute present value estimates, and uses the remaining subperiod 1968-1986 for simulation. He uses the whole set 1953-1986 when applying the historical period approach.

Table 1 shows the estimated present values (EPV), for the four different methods discussed earlier, reported in Dulaney (1987). Near the bottom of the table are historical averages of each EPV. Also included, at the far right, is the simulated present value (SPV). The bottom row of the table reports the discrepancy measure RMSE, between EPV and SPV. As observed by Dulaney (1987), the base period projection method yields a time-averaged EPV closest to the time-averaged SPV. However, note that it is the total offset method that achieves the best *fit* between EPV and SPV, as measured by RMSE.

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<sup>5</sup>In the case of the year 1953, the first three months' data is unavailable, so to get that year's annualized rate I average the monthly rates for April through December.

TABLE 1: Present Values, from Dulaney (1987)

	base	base	historical		simulated
	year	period	period	total	present
year	projection	projection	projection	offset	value
1953	\$215,946		\$197,913	\$200,000	\$224,673
1954	\$224,218		\$197,913	\$200,000	\$225,450
1955	\$305,294	\$244,975	\$197,913	\$200,000	\$225,627
1956	\$280,378	\$267,516	\$197,913	\$200,000	\$219,805
1957	\$207,598	\$260,083	\$197,913	\$200,000	\$215,429
1958	\$237,307	\$239,524	\$197,913	\$200,000	\$216,725
1959	\$190,772	\$210,742	\$197,913	\$200,000	\$215,498
1960	\$197,093	\$207,095	\$197,913	\$200,000	\$218,744
1961	\$221,703	\$202,632	\$197,913	\$200,000	\$221,382
1962	\$206,927	\$208,266	\$197,913	\$200,000	\$221,328
1963	\$234,964	\$220,852	\$197,913	\$200,000	\$221,894
1964	\$194,079	\$211,167	\$197,913	\$200,000	\$218,971
1965	\$265,438	\$229,260	\$197,913	\$200,000	\$219,241
1966	\$203,044	\$218,270	\$197,913	\$200,000	\$212,672
1967	\$267,875	\$243,047	\$197,913	\$200,000	\$210,634
mean	\$230,176	\$227,956	\$197,913	\$200,000	\$219,205
rmse	\$34,795	\$85,008	\$21,722	\$19,681	0

Since Dulaney's work in 1987, the data for hourly wage has been revised by the U.S. Department of Labor. Consequently, when I re-compute the results in Table 1 using the revision that is current as of October 2009, the numbers are substantially different. I report these numbers in Table 2. Notice that with revised data it is the total offset method that has time-averaged EPV closest to the time-averaged SPV, unlike in Dulaney's original data. Also, the total offset method achieves the best fit between EPV and SPV, as indicated by the RMSE statistics.

TABLE 2: Present Values, Based on Revised Data

	base	base	historical		simulated
	year	period	period	total	present
year	projection	projection	projection	offset	value
1953	\$302,778		\$198,130	\$200,000	\$216,182
1954	\$235,733		\$198,130	\$200,000	\$218,238
1955	\$201,600	\$242,647	\$198,130	\$200,000	\$222,860
1956	\$286,918	\$238,565	\$198,130	\$200,000	\$224,687
1957	\$267,837	\$248,920	\$198,130	\$200,000	\$225,164
1958	\$236,267	\$262,727	\$198,130	\$200,000	\$223,843
1959	\$193,033	\$229,791	\$198,130	\$200,000	\$221,719
1960	\$203,989	\$210,053	\$198,130	\$200,000	\$218,365
1961	\$206,981	\$201,204	\$198,130	\$200,000	\$210,747
1962	\$220,555	\$210,352	\$198,130	\$200,000	\$202,806
1963	\$197,866	\$208,212	\$198,130	\$200,000	\$194,705
1964	\$195,003	\$204,080	\$198,130	\$200,000	\$186,172
1965	\$190,309	\$194,356	\$198,130	\$200,000	\$179,400
1966	\$232,848	\$205,131	\$198,130	\$200,000	\$175,826
1967	\$213,392	\$211,391	\$198,130	\$200,000	\$170,305
mean	\$225,674	\$220,572	\$198,130	\$200,000	\$206,068
rmse	\$37,219	\$81,886	\$20,597	\$19,951	0

To make use of more recent data, as in Brush (2004) I repeat Dulaney's experiment using an updated time frame. Brush (2004) points out that Dulaney's selection of a "historical period", in the historical period approach, unrealistically extends beyond the information set available when computing present value estimates. To remedy this I split the total period 1953-2008 as follows: the historical period is 1953-1973, the estimation period is 1974-1988, and the simulation period is 1989-2008.

Table 3 reports results for the updated sample. As in Tables 1 and 2, the total offset approach achieves the smallest RMSE, among the estimated present value methods. The time-average of EPV is closest to that of SPV when EPV is specified as base year projection, in Table 3, which is a result different than those in Tables 1 and 2.

TABLE 3: Present Values, Recent Time Frame

	base	base	historical		simulated
	year	period	period	total	present
year	projection	projection	projection	offset	value
1974	\$239,387	\$227,585	\$221,672	\$200,000	\$152,394
1975	\$263,466	\$244,426	\$221,672	\$200,000	\$149,447
1976	\$241,074	\$247,703	\$221,672	\$200,000	\$148,723
1977	\$226,897	\$243,318	\$221,672	\$200,000	\$148,196
1978	\$207,340	\$224,507	\$221,672	\$200,000	\$154,325
1979	\$197,375	\$209,979	\$221,672	\$200,000	\$156,856
1980	\$183,965	\$195,825	\$221,672	\$200,000	\$163,719
1981	\$129,815	\$166,461	\$221,672	\$200,000	\$169,444
1982	\$121,407	\$141,792	\$221,672	\$200,000	\$173,617
1983	\$114,005	\$121,614	\$221,672	\$200,000	\$184,356
1984	\$103,061	\$112,510	\$221,672	\$200,000	\$188,881
1985	\$127,474	\$114,201	\$221,672	\$200,000	\$191,303
1986	\$166,122	\$128,573	\$221,672	\$200,000	\$191,683
1987	\$137,056	\$142,275	\$221,672	\$200,000	\$193,470
1988	\$150,420	\$150,589	\$221,672	\$200,000	\$196,231
mean	\$173,924	\$178,090	\$221,672	\$200,000	\$170,843
rmse	\$66,719	\$67,192	\$53,914	\$34,253	0

## 6 Discussion

To better understand the reported differences among EPV methods, in terms of root mean squared error (RMSE), let  $Z$  represent the gap between EPV and SPV values:

$$Z_i = EPV_i - SPV_i$$

during each period  $i$ . Then the mean square error (MSE) statistic takes the form:

$$MSE = \frac{1}{n} \sum_{i=1}^n Z_i^2$$

Taking the square root of MSE yields RMSE, and we can express the former as:

$$MSE = |\bar{Z}|^2 + S_Z^2 \tag{6}$$

where:

$$\bar{Z} = \frac{1}{n} \sum_{i=1}^n Z_i$$

$$S_Z^2 = \frac{1}{n} \sum_{i=1}^n (Z_i - \bar{Z})^2$$

In equation 6 the expression  $|\bar{Z}|$  estimates the absolute bias of estimated PV for simulated PV.  $S_Z$  is an estimate of standard deviation for the gap  $Z$  between  $EPV$  and  $SPV$ . MSE is increasing in absolute bias, and also in standard deviation.

Table 4 shows absolute bias and standard deviation (Std Dev) statistics, for the various samples discussed earlier. Since total offset achieves the lowest RMSE in each case, it follows that it must achieve a lower absolute bias, or lower error standard deviation, when compared to any other EPV method. As it turns out no one EPV method dominates the others in terms of bias, but Table 4 shows that both the total offset and historical period approaches achieve near-identical values of error standard deviation, these being lower than those of the remaining two methods. This, combined with the fact that total offset achieves smaller absolute bias than does the historical period approach, makes the total offset approach superior in terms of mean squared error.

The results in Tables 1 through 4 are based on two empirical examples. For a third example, I consider the data studied by Brush (2004): the data definitions are the same as here and in Dulaney (1987), but the estimation period is 1968-1982. For the historical period approach, the historical period is 1953-1967. The bottom-most block of Table 4 reports on this example: again the total offset approach achieves the smallest RMSE, and the results are generally similar to the cases already discussed.

TABLE 4: Error Decomposition

estimation	EPV			
period	method	RMSE	Bias	Std Dev
1953-1967 (Dulaney)	base year	\$34,795	\$10,971	\$33,020
	base period	\$85,008	\$8,751	\$84,556
	historical period	\$21,722	\$21,292	\$4,301
	total offset	\$19,681	\$19,205	\$4,302
1953-1967 (revised)	base year	\$37,219	\$19,606	\$31,636
	base period	\$81,886	\$14,504	\$80,591
	historical period	\$20,597	\$7,938	\$19,006
	total offset	\$19,951	\$6,068	\$19,006
1974-1988	base year	\$66,719	\$3,081	\$66,648
	base period	\$67,192	\$7,247	\$66,800
	historical period	\$53,914	\$50,829	\$17,976
	total offset	\$34,253	\$29,157	\$17,976
1968-1982 (Brush)	base year	\$67,446	\$50,552	\$44,648
	base period	\$65,567	\$55,967	\$34,157
	historical period	\$65,073	\$64,670	\$7,231
	total offset	\$42,495	\$41,875	\$7,233

Before investing too heavily in the results reported in Tables 1 through 4, it is important to note that Dulaney’s (1987) definition of simulated present value differs from those considered by later authors, particularly Palaez (1991) and Brush (2004), for whom the relevant formula is:

$$SPV^* = \sum_{i=1}^n Y \left( \frac{1 + W_i^*}{1 + R_i^*} \right)^i \quad (7)$$

where:

$$W_i^* = \left( \prod_{j=1}^i (1 + W_j) \right)^{1/i} - 1 \quad (8)$$

$$R_i^* = \left( \prod_{j=1}^i (1 + R_j) \right)^{1/i} - 1 \quad (9)$$

$SPV^*$  represents a “fair award” or “actual” present value. In a hypothetical world where wages and interest rates are non-stochastic, such that all future values of  $W$  and  $R$  are known at any given date,  $SPV^*$  is the unique arbitrage-free value of the  $n$ -period annuity income stream  $Y, \dots, Y$ .

Table 5 reports results based on the alternative definition  $SPV^*$  of simulated present value.<sup>6</sup> While these results are numerically different than those in Table 4, they are qualitatively similar: again the total offset approach achieves the best fit in terms of RMSE. It also has the smallest absolute bias, in each reported example, and is essentially tied with the historical

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<sup>6</sup>The case of Dulaney’s (1987) data is omitted here as he did not report  $W_i$  and  $R_i$  for periods  $i = 1968 - 1986$ , these being needed to calculate  $W_i^*$  and  $R_i^*$ .

period method for smallest error standard deviation.

TABLE 5: Error Decomposition, Alternative *SPV*

estimation	EPV			
sample	method	RMSE	Bias	Std Dev
1953-1967	base year	\$34,954	\$11,853	\$32,883
	base period	\$82,921	\$6,751	\$82,646
	historical period	\$16,315	\$15,691	\$4,470
	total offset	\$14,526	\$13,821	\$4,472
1974-1988	base year	\$54,400	\$21,490	\$49,975
	base period	\$57,478	\$25,656	\$51,434
	historical period	\$70,096	\$69,238	\$10,934
	total offset	\$48,806	\$47,566	\$10,932
1968-1982	base year	\$55,799	\$44,213	\$34,040
	base period	\$57,144	\$49,628	\$28,328
	historical period	\$62,356	\$58,331	\$22,040
	total offset	\$41,816	\$35,536	\$22,040

In the foregoing have focused on RMSE as a measure of forecast accuracy. If instead I apply mean absolute error (MAE), the results are very similar: in 5 out of 6 cases the total offset approach achieves the smallest MAE, hence is the most accurate forecast of simulated present value. The one exception is for the up-to-date data with estimation sample period 1974-1988 and with

simulated present value defined as  $SPV^*$ : here MAE equals \$47,566 for total offset, while minimum MAE is achieved by the base year approach, with MAE = \$46,483. But even here, total offset comes within 2 percent of the best achievable MAE.

## 7 Related Results

The simulation excersises, reported earlier, describe the match between ex post present value and various an ex ante estimation methods. The empirical success of the total offset method, whereby estimates of future net discount rates  $G$  are each set to zero, is consistent with the the hypothesis that the  $G$  fluctuates around the value 0 in the long run. A related exercise is therefore to test the null hypothesis:

$$H_0: \mu_G = 0$$

where  $\mu_G$  is the mean value of  $G$ , under the assumption that  $G$  is a stationary stochastic process. The two-sided alternative to  $H_0$  is that  $\mu_G$  is non-zero.

Figure 1 shows the time path of the net discount rate  $G$ , for the years 1953-2008. As indicated,  $G$  was mostly positive until 1980, mostly negative from 1981-1999. From year 2000 onward  $G$  appears to fluctuate around the value 0. Over the whole sample period 1953-2008 the central tendency of  $G$  is somewhat ambiguous, as the sample average  $G$  is negative (-1.96 percent) while the median value is positive (0.01 percent).

Table 6, fist row, reports the Student's  $t$  statistic for  $H_0$ , as well as a p-

value, under the assumption that  $G$  is independent and identically distributed normal. As the  $p$ -value exceeds 0.10, evidence against  $H_0$  is not significant at standard levels. This is consistent with the idea that  $G$  fluctuates around 0 in the long run. The remaining rows of Table 6 report results for tests robust to serial correlation in  $G$ . The first of these, labeled “autoregression,” is based on an autoregressive AR(2) model:<sup>7</sup>

$$G_t = \alpha + \beta_1 G_{t-1} + \beta_2 G_{t-2} + \varepsilon \quad (10)$$

For a stationary AR(2) process the mean value is:<sup>8</sup>

$$\mu_G = \frac{\alpha}{1 - (\beta_1 + \beta_2)}$$

hence the hypothesis  $H_0$  is equivalent to the restriction  $\alpha = 0$ . Row 2 of Table 1 reports the  $t$ -test for this intercept restriction in the regression model of  $G_i$  on  $G_{i-1}$  and  $G_{i-2}$ . As indicated, the AR(2) adjustment for autocorrelation raises the  $p$ -value relative to the classic random-sample  $t$ -test, hence  $H_0$  is again not rejected.

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<sup>7</sup>Sample autocorrelations, for  $G$ , are: 0.69, 0.56, 0.36 for lags 1, 2, 3, respectively.

<sup>8</sup>I assume that the stationarity AR(2) model applies to  $G_i$  at all integer dates  $i = \dots - 1, 0, 1, \dots$ . See Enders (2010, p. 61) for discussion.

TABLE 6: Tests for Non-zero Mean in Net Discount Rate

test method	test statistic	p-value
student's t	-1.54	0.13
autoregression	-0.15	0.88
z (Bartlett)	-0.88	0.38
z (block bootstrap)	-0.26	0.79

If  $G$  is not necessarily an AR(2) process but is nevertheless stationary and mean-reverting then, under fairly general conditions, the sample average  $\bar{G}$  will converge to the mean value  $\mu_G$  when the sample size  $T$  gets large, and the deviation  $\bar{G} - \mu_G$  will be normally distributed. More precisely:

$$\frac{\bar{G} - \mu_G}{\sqrt{T}} \xrightarrow{d} \text{Normal}(0, \lambda) \quad (11)$$

where  $\xrightarrow{d}$  means “converge in distribution” and  $\lambda$  is the long-run variance of  $G$ . If a consistent estimate  $\hat{\lambda}$  is available for  $\lambda$ , then the  $z$  statistic:

$$z = \frac{\bar{G}}{\sqrt{\hat{\lambda}}} \quad (12)$$

is distributed as standard normal in large samples, under  $H_0$ . Because  $G_t$  is allowed to exhibit serial correlation,  $\hat{\lambda}$  must embody information about the sample autocorrelations of  $G$ . Row 3 and 4 of Table 6 report two versions of the  $z$  test: in the first the long-run variance estimate  $\hat{\lambda}$  is that of Bartlett (1946), and the second is based on a version of Monte Carlo simulation called

block bootstrap (see Hall 1992).<sup>9</sup> Both of these  $z$  tests fail to reject  $H_0$ , in agreement with the other tests.

The failure to reject  $H_0$  can be viewed as result of a relatively weak “signal” (mean value) relative to “noise” (standard error). In principle, the noise might be reduced by using higher frequency data. To this end I run an autoregressive model on quarterly data – this being the highest frequency at which both the wage and interest rate data are available. I include 8 quarterly lags, to cover 2 years of dynamics, matching the annual AR(2) reported earlier. The  $t$  test for zero intercept in the AR(8) autoregression has statistic  $t = -1.65$ , and p-value 0.0998, barely significant at the 10 percent level. Hence, recourse to higher frequency data does not cast strong doubt on  $H_0$ .

All of the afore-mentioned tests assume stationarity of the net discount rate, but  $G$  might be non-stationarity. As a check, for the period 1953-2008 I find that standard tests favor stationarity when pitted against the unit root form of non-stationarity.<sup>10</sup> This is generally consistent with results reported previously, see Brush (2004) for a review.

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<sup>9</sup>For the Bartlett method,  $\hat{\lambda}$  is a weighted average of sample autocovariances of  $G_t$ , with weights that decline linearly at longer lags, equaling zero beyond some maximum lag  $L$  which I set equal to 3. For the block bootstrap, I use Monte Carlo simulation to resample  $G$  values (with replacement) in time blocks of length 5 years, then compute  $\hat{\lambda}$  as the the variance of the resampled statistic.

<sup>10</sup>The augmented Dickey-Fuller test of the unit root null hypothesis has p-value = 0.0447, rejecting a unit root in favor of stationarity, at the 5 percent level. The Phillips-Perron test result is nearly identical, with p-value = 0.0459. For testing the stationarity null hypothesis versus the unit root alternative, the Kwiatkowski-Phillips-Schmidt-Shin statistic equals 0.3399, not significantly rejecting stationarity at the 10 percent level.

Another possibility is structural change: perhaps there was a permanent shift in the level of  $G$  at some point in the past. Johnson and Gelles (1996) and Payne, Ewing and Piette (1999) argue that the level of  $G$  may have shifted sometime around the year 1980, from positive to negative values. Soaring inflation in the 1970s was quickly absorbed into nominal interest rates, but not so quickly absorbed into nominal wages, thus causing  $G$  to swing into negative values. To model this formally, suppose that the level of  $G$  undergoes repeated shifts, or “switching regimes”, and consider the stationary switching regime AR(1) model studied by Hamilton (1989):

$$G_t = \alpha_t + \beta G_{t-1} + \varepsilon_t \tag{13}$$

where  $\alpha_t$  is a time-varying intercept that switches between two possible values, “high” and “low” over time, as a 2-state Markov chain. Here the level  $\mu_G$  of  $G_t$  is time-varying and equal to  $\alpha_t/(1 - \beta)$ . Estimating this model via Gaussian maximum likelihood, the fitted values of  $\mu_G$  in high and low states are 2.22 and -3.39, respectively. The estimated probability of being in the high state in the year 2009 equals 0.9998. In other words, the fitted model assigns near certainty to situation where  $G$  is currently fluctuating around a positive level, in sharp contrast to the idea that  $G$  shifted to a permanent negative level in some year around 1980.

What level (negative, positive, or zero) should be assigned to the net discount rate for the next 20 years? If the U.S. is hit by a sustained period of

soaring oil prices then  $G$  should run negative: otherwise  $G$  may run positive. As discussed in Palaez (1991), various other economic factors are relevant, including the future course of labor productivity, personal savings, and budget deficits. Palaez (1991) used published projections of such variables to predict a rise in  $G$  in the 1990s, relative to the 1980s, and he was right. Furthering this idea, one could apply a full-scale macroeconomic model to make use of factors affecting  $G$ , as in the Fair model<sup>11</sup>. However, oil prices are key to the sign of  $G$ , and predicting swings in oil prices is notoriously difficult. For this reason, and those given earlier, the hypothesis of zero level for future  $G$  seems plausible.

## 8 Conclusion

Evidence suggests that the total offset approach has a consistent advantage over competing methods, as an estimate of the present value of future earnings. With total offset there is superior agreement between ex ante estimated present value and ex post simulated present value. The result holds up over three different sample periods, and two definitions of “simulated” present value. The analysis is based on aggregate wage data: in the future it would be useful to see a similar analysis for specific industries.

Hopefully I have convinced the reader that, if one attempts to compare an ex ante present value to an ex post simulated present value, it pays to

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<sup>11</sup>due to Ray Fair, see [fairmodel.econ.yale.edu](http://fairmodel.econ.yale.edu)

explore the meaning of such a comparison. The very concept of *simulated* or *fair* or *actual* present value, as envisioned by Dulaney (1987), Palaez (1991), and Brush (2004), could use more discussion in the literature. These are technical points, but the general forensic economist knows the importance of methodology in the estimation of present values. Better methods, or at least a better understanding of existing methods, can go some way to helping the profession in its effort to value the loss of future income due to death and disability.

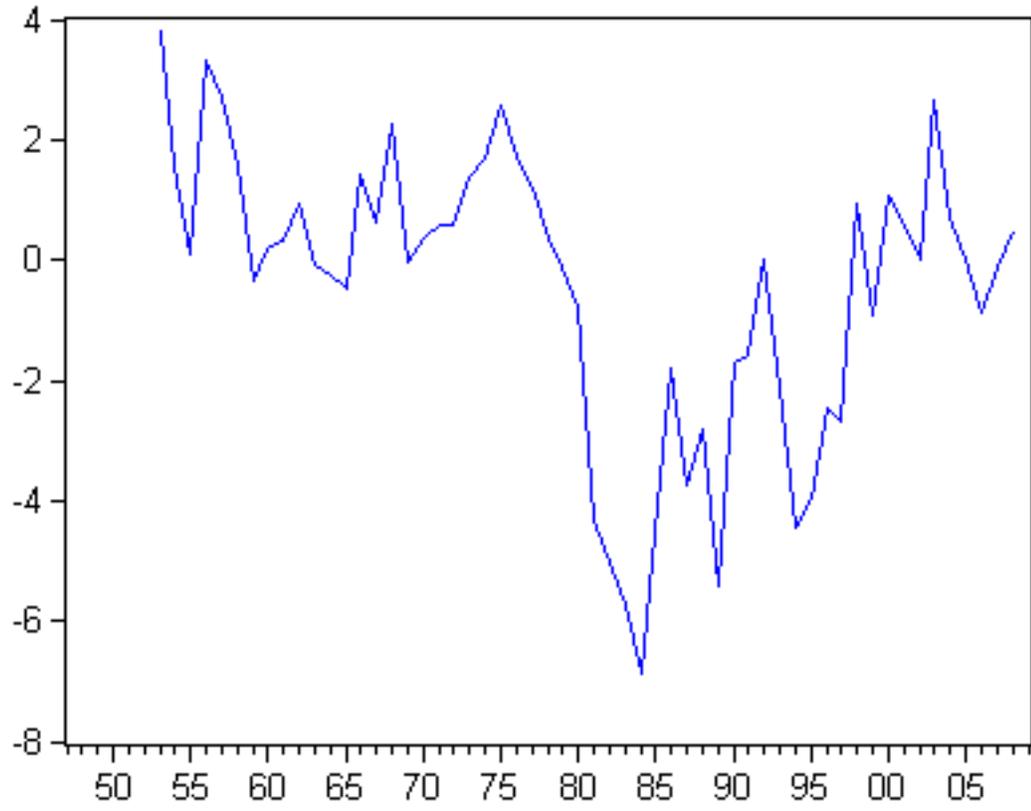


Figure 1: Plot of Net Discount Rate ( $G$ )

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