Tax and the Present Value of Future Income

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Overview

- Impact of tax on income lost to injury or death.
- Simple case: constant income stream.
- More (FE) relevant: growing income streams.
- Anderson and Barber (2010) tax model.
- Problem in model, a solution.
- Main result: tax effect (+ or -) depends on expected worklife.

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- Journal of Forensic Economics (1994)
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Economics:

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Death and Taxes

Situation: injury or death at time 0

Lost future incomes: $E_1, E_2, ...$

Present value: P_t

Interest rate: r

Tax rate: τ

Future Value equation:

$$(1 + r)P_t = (1 - \tau)E_{t+1} + \tau r P_t + P_{t+1} \tag{1}$$

Perpetuity

Constant future earnings $E_1 = E_2 = \ldots$

Present value:

$$P = \frac{E}{r} \tag{2}$$

- Damages P invariant to tax rate τ.
- Reason: constant price P_t over time, so from FV:

$$(1 + r)P = (1 - \tau)E + \tau r P + P \tag{3}$$

- Economics literature: Modigliani and Miller (1963)
- Conclusion: no tax effect on PV of perpetual income.

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Annuity

Constant future earnings $E_1 = E_2 = \ldots = E_N$

Present value:

$$P = \frac{E}{r} \left(1 - \frac{1}{(1 + (1 - \tau)r)^N}\right) \tag{4}$$

- Damages P decreasing in tax rate τ.
- Reason: FV and $P_N = 0$
- Economics literature: Samuelson (1964)
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Growing Income

Incomes: $E_1, E_2, ..., E_N$

Present value, at time 0:

$$P_0 = \sum_{t=1}^{N} \frac{(1-\tau)E_t}{(1+r(1-\tau))^t} \tag{5}$$

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Anderson and Barber (2010)

Constant income growth:

$$E_t = E_0(1+g)^t \tag{6}$$

$E_0 =$ base income
$g =$ growth rate

Present value:

$$PV_{earningsaftertax} = \frac{E_0(1-\tau^e)(1+g)}{(r(1-\tau^i)-g)}\left[1-\left(\frac{1+g}{1+r(1-\tau^i)}\right)^N\right] \tag{7}$$

$$PV_{earningswithouttax} = \frac{E_0(1+g)}{(r-g)}\left[1-\left(\frac{1+g}{1+r}\right)^N\right] \tag{8}$$

Mixed Effects of Tax

Anderson and Barber "breakeven point":

$$\tau^e = -\frac{\tau^i r D}{(1+r)} \tag{9}$$

where D is the constant:

$$D = -\frac{(1+r)}{(r-g)} + \frac{N\left(\frac{1+g}{1+r}\right)^N}{\left[1-\left(\frac{1+g}{1+r}\right)^N\right]} \tag{10}$$

- For small N, LH $>$ RH and tax effect < 0
- For large N, LH $<$ RH and tax effect > 0
- Conclusion: tax lowers economics damage if worklife short, otherwise raises it.

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Example

$\tau^e = \tau^i = 0.1$

$g = r = 0.05$

$$PV_{earningswithouttax} = \sum_{t=1}^{N} \frac{E_0(1+g)^i}{(1+r)^i} \tag{11}$$

and:

$$D = -\frac{\sum_{t=1}^{N} \frac{tE_0(1+g)^t}{(1+r)^t}}{\sum_{t=1}^{N} \frac{E_0(1+g)^t}{(1+r)^t}} \tag{12}$$

Breakeven condition: $D = -21$, $N = 40$.

- Tax lowers PV if $N < 40$, raises it if $N > 40$.
- Or does it?

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- Or does it?

Example, details

Table: Present values, Tax Rate = 10 percent

N	PV without tax	PV with tax
39	39	38.67
40	40	39.76
41	41	40.86
42	42	41.96
43	43	43.06
44	44	44.17

Other Examples

Table: Present values, Tax Rate = 1 percent

N	PV without tax	PV with tax
39	39	38.980
40	40	39.989
41	41	40.999
42	42	42.009
43	43	43.019
44	44	44.030

Table: Present values, Tax Rate = 50 percent

N	PV without tax	PV with tax
39	39	32.75
40	40	34.06
41	41	35.40
42	42	36.78
43	43	38.19
44	44	39.63
45	45	41.11
46	46	42.63
47	47	44.18
48	48	45.77
49	49	47.40
50	50	49.06
51	51	50.77
52	52	52.52

Breakeven Possibilities

Table: Breakeven Possibilities

tax rate	0.01	0.05	0.1	0.2	0.3	0.4	0.5
breakeven point	41	42	43	44	47	49	52

Forensics

Macaulay's duration (Maucaulay 1938, Hicks 1939): a weighted average of income arrival dates $1, 2, ..., N$.

For a bond, coupon I, face value F:

$$D = \frac{\frac{I}{R} + \frac{2I}{R^2} + \cdots + \frac{NI}{R^N} + \frac{NF}{R^N}}{\frac{I}{R} + \frac{I}{R^2} + \cdots + \frac{I}{R^N} + \frac{F}{R^N}} \qquad (13)$$

Generally:

$$D = \sum_{t=1}^{N} t \frac{PV_t}{PV} \qquad (14)$$

$$PV_t = \frac{E_t}{(1+r)^t} \qquad (15)$$

Macaulay's duration is then:

$$D = \frac{\sum_{t=1}^{N} \frac{tE_t}{(1+r)^t}}{\sum_{t=1}^{N} \frac{E_t}{(1+r)^t}} \tag{16}$$

Hick's interpretation: write PV as:

$$PV = \sum_{t=1}^{N} R^{-t} E_t \tag{17}$$

with $R =$ gross interest rate.

D is elasticity of PV with respect to R:

$$D = \frac{\partial PV}{\partial R} \frac{R}{PV} \tag{18}$$

- Anderson & Barber's D is negative, Macaulay's duration is positive.
- Doesn't resolve the modelling problem.

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More Forensics

If the negative effect of income tax just offsets the positive effect of interest tax on present value then, at the margin, the change dPV in present value equals 0:

$$dPV = \frac{\partial PV}{\partial \tau^e} d\tau^e + \frac{\partial PV}{\partial \tau^i} d\tau^i = 0 \tag{19}$$

Suppose, moreover, that the earnings tax rate τ^e grows at the same instantaneous rate as does the tax rate τ^i on interest:

$$\frac{d\tau^e}{\tau^e} = \frac{d\tau^i}{\tau^i} \tag{20}$$

Then we can then interpret the marginal condition (19) on present value in terms of elasticities:

$$\frac{\partial PV}{\partial \tau^e} \frac{\tau^e}{PV} = -\frac{\partial PV}{\partial \tau^i} \frac{\tau^i}{PV} \tag{21}$$

The elasticity of (after-tax) PV with respect to the earnings tax rate is:

$$\frac{\partial PV}{\partial \tau^e}\frac{\tau^e}{PV} = -\frac{\tau^e}{1-\tau^e} \tag{22}$$

For a small tax rate τ_e on earnings, we can approximate this elasticity as follows:

$$\frac{\partial PV}{\partial \tau^e}\frac{\tau^e}{PV} \approx -\tau_e \tag{23}$$

Elasticity of PV with respect to the interest tax rate:

$$\frac{\partial PV}{\partial \tau^i}\frac{\tau^i}{PV} = \frac{\partial PV}{\partial(1+r(1-\tau^i))}\frac{\partial(1+r(1-\tau^i))}{\partial \tau^i}\frac{\tau^i}{PV} \quad (24)$$

$$= -\frac{r\tau_i}{1+r(1-\tau^i)}\frac{\partial PV}{\partial(1+r(1-\tau^i))}\frac{1+r(1-\tau^i)}{PV} \quad (25)$$

$$= \frac{r\tau_i}{1+r(1-\tau^i)}D_{aftertax} \quad (26)$$

with $D_{aftertax}$ the variant of Macaulay-Hicks duration D based on after-tax earnings and interest:

$$D_{aftertax} = \frac{\sum_{t=1}^{N}\frac{tE_t}{(1+(1-\tau^i)r)^t}}{\sum_{t=1}^{N}\frac{E_t}{(1+(1-\tau^i)r)^t}} \quad (27)$$

If the tax rate τ^i is close to zero then after-tax duration is about the same as before-tax duration:

$$D_{aftertax} \approx D \quad (28)$$

If tax rates τ^i and τ^e are both small then, applying the small-tax approximations (23) and (28), the balancing condition (19) is approximately:

$$\tau^e \approx \frac{\tau^i r}{1+r} D \quad (29)$$

Interpretation:

- Anderson and Barber breakeven condition (9) restated as a small-tax approximation.
- A minus (-) sign appears on the right-hand side of (9) but not (29). This difference in sign reflects the difference between the Anderson and Barber constant D and Macaulay-Hicks duration D.

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Small-Tax Theory

$$\frac{\partial}{\partial \tau} PV_{earningsaftertax} = \sum_{t=1}^{N} \frac{-(1+g)^t}{(1+r(1-\tau))^t} + \sum_{t=1}^{N} \frac{tr(1-\tau)(1+g)^t}{(1+r(1-\tau))^{t+1}} \quad (30)$$

Evaluating the derivative at $\tau = 0$, and denoting the result as PV', yields:

$$PV' = \sum_{t=1}^{N} \left(t\frac{r}{1+r} - 1 \right) \left(\frac{1+g}{1+r} \right)^t \quad (31)$$

$$PV' = \frac{r}{1+r}\sum_{t=1}^{N} t a^t - \sum_{t=1}^{N} a^t \quad (32)$$

$$= \frac{r}{1+r}\frac{a}{1-a}\left(\frac{1-a^N}{1-a} - Na^N\right) - \frac{a(1-a^N)}{1-a} \quad (33)$$

where:

$$a = \frac{1+g}{1+r} \quad (34)$$

Proposition:

Suppose that an earnings stream grows at a constant positive rate, and that earnings and interest are taxed at equal rates. Let U be any finite upper bound for the earnings horizon N, such that $U > N^*$ with N^* the closest integer approximation to the breakeven condition. Then, for all tax rates sufficiently small, the present value of the earnings stream is lower after-tax than without tax when horizon N is less than N^*, but is higher after tax when N is greater than N^* yet less than U.